PROBLEM SHEET ON CURVES IN EUCLIDEAN SPACE AND COSSERAT FRAMES

Problem 1. *Consider the parameterized curve*

$$\alpha(t) = (\frac{1}{3}(\sqrt{1+t})^3, \frac{1}{3}(\sqrt{1-t})^3, \frac{1}{\sqrt{2}}t)$$

which is defined for $t \in [-1, 1]$.

- *Recall from Problem sheet 4, Problem 1, the tangent, normal and binormal vector of* α*;*
- *Write the Frenet-Serret equations;*
- *Find the twist vector of* α.

Problem 2. *Consider the parameterized curve*

$$\beta(t) = (a\cosh(t), a\sinh(t), bt)\,.$$

- *Check if* β *is parameterized by arclength;*
- *Find the normal vector and the curvature of* β *for every* $t \in \mathbb{R}$*;*
- *Find the center of the osculating circle for every* $t \in \mathbb{R}$*;*
- *Find the binormal vector and the torsion of* β *for every* $t \in \mathbb{R}$*;*

Draw some pictures of the curve.

Problem 3. *Let* $b > 0$ *and consider the three-dimensional body defined by:*

$$\mathcal{B} = \{(x, y, z) : 0 \leq x \leq b, e^{-x} \leq y \leq e^{x}, 0 \leq z \leq 2e^{x}\},$$

and assume the sets $\{x = a\}$ *(for* $a \in \mathbb{R}$ *constant) are its cross sections.*

- *Draw a picture of the rod and of its cross sections;*
- *Find the line of centroids of the rod.*

Problem 4. *Consider the curve* $r(t) = (t, 0, e^t)$ *and the triple*

$$d_1(t) = (\cos(e^t), \sin(e^t), 0) \quad d_2(t) = (-\sin(e^t), \cos(e^t), 0) \quad d_3 = (0, 0, 1)$$

- *Check that* (d_1, d_2, d_3) *is an orthonormal triple;*
- *Find coefficients* v_1, v_2, v_3 *such that* $r'(t) = v_1 d_1 + v_2 d_2 + v_3 d_3$*;*
- *Find the vector* u *such that, for* $k = 1, 2, 3$, $d_k'(t) = u \times d_k$.
- *Check that the Cosserat frame preserves orientation, i.e.* $r'(t) \cdot d_3 > 0$.

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