EXERCISE SHEET 10

WRITTEN SOLUTIONS OF EXERCISES 1.6 AND 1.12 TO BE PRESENTED ON 11/12

Exercise 1. For each of the following functions f, determine the intervals on which f is convex and concave. Sketch a picture of the graph of f, by studying appropiately the domain of definition of f, the limits of f, and the intervals of monotonicity .

(1) $f(x) = x^2 - 6x + 5$.
(2) $f(x) = -2x^2 + 4x - 7$.
(3) $f(x) = 2x^2 + x - 6$.
(4) $f(x) = x^3 - 9x^2 + 1$.
(5) $f(x) = x^4 - 2x^3 - 7$.
(6) $f(x) = x^4 - x^5$.
(7) $f(x) = x^4 + 2x^3 + 6x^2$.
(8) $f(x) = x + \frac{1}{x}$.
(9) $f(x) = \sinh(x)$.
(10) $f(x) = \cosh(x)$.
(11) $f(x) = \tanh(x)$.
(12) $f(x) = \frac{x-1}{x+1}$.
(13) $f(x) = \frac{3x^2}{1-x}$.
(14) $f(x) = \frac{2x}{1+x^2}$.
(15) $f(x) = \frac{x^2+1}{x^2-1}$.
(16) $f(x) = \frac{2x}{\sqrt{x^2+1}}$.
(17) $f(x) = \sqrt{1-x}$.
(18) $f(x) = xe^{-x}$.
(19) $f(x) = \ln(x-1)$.
(20) $f(x) = e^{-x^2}$.

Exercise 2. The goal of this exercise is to prove Lagrange's theorem, based on Rolle's theorem. Recall that Rolle's theorem says that if $g : I \to \mathbb{R}$ is differentiable and $g(a) = g(b)$ for $a < b$, $a, b \in I$, then there exists a point $y \in (a, b)$ such that $g'(y) = 0$

(1) Given a differentiable function $f : I \to \mathbb{R}$, write the equation of the unique line $\ell : \mathbb{R} \to \mathbb{R}$ which passes through the points $(a, f(a))$ and $(b, f(b))$.
(2) Let $g = f - \ell$. Check that $g(a) = g(b) = 0$.
(3) Apply Rolle's theorem to g to show that there exists $y \in (a, b)$ such that $f'(y) = \ell'(y)$.
(4) Conclude that
$$f'(y) = \frac{f(b) - f(a)}{b - a} \ .$$