EXERCISE SHEET 3

WRITTEN SOLUTIONS OF EXERCISES 1.2, 2.2 AND 3.2 TO BE PRESENTED ON 16/10

Exercise 1. Give the following definitions of limits of a function $f : \mathbb{R} \to \mathbb{R}$:

(1) $\lim_{x \to +\infty} f(x) = -\infty$;

(2) $\lim_{x \to -\infty} f(x) = \ell \in \mathbb{R}$;

(3) $\lim_{x \to x_0} f(x) = \ell \in \mathbb{R}$;

(4) $\lim_{x \to x_0^+} f(x) = +\infty$;

(5) $\lim_{x \to x_0^-} f(x) = +\infty$;

(6) $\lim_{x \to x_0^+} f(x) = \ell \in \mathbb{R}$;

(7) $\lim_{x \to x_0^-} f(x) = -\infty$.

Exercise 2. By applying the definition, prove the following limits:

(1) $\lim_{x \to -\infty} \frac{1}{x} = 0$;

(2) $\lim_{x \to +\infty} \frac{1}{\sqrt{x}} = 0$;

(3) $\lim_{x \to 0^+} \frac{1}{x} = +\infty$;

(4) $\lim_{x \to 0} \frac{1}{x^2} = +\infty$;

Exercise 3. Determine if the following limits exist and, if they exist, compute the value of the limit.

(1) $\lim_{x \to +\infty} \left(1 + \frac{1}{x}\right)$;

(2) $\lim_{x \to +\infty} \left(\frac{1}{x} - \frac{1}{2x}\right)$;

(3) $\lim_{x \to 0} f(x)$ where $f(x) = \begin{cases} 1 & \text{if } x = 1/n \text{ for } n \in \mathbb{N} \\ 0 & \text{otherwise} \end{cases}$;

(4) $\lim_{x \to 0} f(x)$ where $f(x) = \begin{cases} 1/n^2 & \text{if } x = 1/n \text{ for } n \in \mathbb{N} \\ 0 & \text{otherwise} \end{cases}$;