EXERCISE SHEET 4

WRITTEN SOLUTIONS OF EXERCISES 1.20, 2, 3.3 AND 4 TO BE PRESENTED ON 23/10

Exercise 1. Determine if the following limits exist, and if so, compute their value:

(1) $\lim_{x\to+\infty} \frac{x^2+1}{3x-2}$;

(2) $\lim_{x\to+\infty} \frac{x^4+3x^3+7}{(x^2+2)(2-x^2)}$;

(3) $\lim_{x\to-\infty} \frac{x^3+x+4}{x^2-x+5}$;

(4) $\lim_{x\to-\infty} \frac{-2x^2+1}{3x^2-6}$;

(5) $\lim_{x\to-\infty} \frac{-2x^2+1}{3x^2-6}$;

(6) $\lim_{x\to+\infty} \frac{2xe^x}{4x^2-6}$;

(7) $\lim_{x\to-\infty} \frac{2xe^x}{4x^2-6}$;

(8) $\lim_{x\to+\infty} \frac{(x^2+3)(x^3-7)}{(x-1)(x^3+9)}$;

(9) $\lim_{x\to-\infty} \frac{(x^2+3)(x^3-7)}{(x-1)(x^3+9)}$;

(10) $\lim_{x\to+\infty} \frac{(x^2+3)(x^3-7)}{(1-x)(x^3+9)}$;

(11) $\lim_{x\to+\infty} e^{-x}\sin x$;

(12) $\lim_{x\to-\infty} \left(1+e^x\cos(x^2)\right)$;

(13) $\lim_{x\to+\infty} \left(1+e^x\cos(x^2)\right)$;

(14) $\lim_{x\to2^+} \frac{x^2+1}{x-2}$;

(15) $\lim_{x\to2^-} \frac{x^2+1}{x-2}$;

(16) $\lim_{x\to2} \frac{x^2+1}{x-2}$;

(17) $\lim_{x\to2^+} \frac{x^2-4}{x-2}$;

(18) $\lim_{x\to2^-} \frac{x^2-4}{x-2}$;

(19) $\lim_{x\to2} \frac{x^2-4}{x-2}$;

(20) $\lim_{x\to4} \frac{x^4+3x^3+7}{(x-4)^2(2-x^2)}$;

(21) $\lim_{x\to+\infty} -\frac{2x^7e^{-x}}{x^3-27}$;

(22) $\lim_{x\to-\infty} -\frac{2x^7e^{-x}}{x^3-27}$;

(23) $\lim_{x\to3^-} -\frac{2x^7e^{-x}}{x^3-27}$;

(24) $\lim_{x\to3^+} -\frac{2x^7e^{-x}}{x^3-27}$;

(25) $\lim_{x\to-1} \frac{x^4-2x^2+1}{(x-1)(x+1)}$;

(26) $\lim_{x\to1} \frac{x^4-2x^2+1}{(x-1)(x+1)}$;

(27) $\lim_{x\to1^-} \frac{x^4+2x^2+1}{(x-1)(x+1)}$;

(28) $\lim_{x\to1^+} \frac{x^4+2x^2+1}{(x-1)(x+1)}$;

(29) $\lim_{x\to+\infty} \frac{x^4+2x^2+1}{(x-1)(x+1)}$;

(30) $\lim_{x\to0} \frac{\tan(x)}{x}$;

(31) $\lim_{x\to0} x\sin\left(\frac{1}{x}\right)$;

(32) $\lim_{x\to0} \sqrt{x}\sin\left(\frac{1}{x}\right)$;

(33) $\lim_{x\to+\infty} \frac{e^{e^x}}{e^x}$.

Exercise 2. By using that

$$\lim_{x\to0}\frac{\sin(x)}{x}=1$$

and the properties of trigonometric functions, show that

$$\lim_{x\to0}\frac{1-\cos(x)}{x^2}=\frac{1}{2}\ .$$

(*Hint: multiply numerator and denominator by* $1+\cos(x)$.)

Then show that

$$\lim_{x\to0}\frac{1-\cos(x)}{x}=0\ .$$

Exercise 3. Find examples of functions $f : \mathbb{R} \setminus \{1\} \to \mathbb{R}$ satisfying the following properties:

(1) $\lim_{x \to +\infty} f(x) = 0,$ $\lim_{x \to -\infty} f(x) = 0$ and $\lim_{x \to 1} f(x) = +\infty;$

(2) $\lim_{x \to +\infty} f(x) = +\infty,$ $\lim_{x \to -\infty} f(x) = -\infty,$ $\lim_{x \to 1^+} f(x) = +\infty$ and $\lim_{x \to 1^-} f(x) = -\infty;$

(3) $\lim_{x \to +\infty} f(x) = -\infty,$ $\lim_{x \to -\infty} f(x) = +\infty$ and $\lim_{x \to 1} f(x) = -\infty;$

(4) $\lim_{x \to +\infty} f(x) = +\infty,$ $\lim_{x \to -\infty} f(x) = -\infty,$ $\lim_{x \to 1^+} f(x) = +\infty$ and $\lim_{x \to 1^-} f(x) = -\infty;$

(5) $\lim_{x \to +\infty} f(x) = -\infty,$ $\lim_{x \to -\infty} f(x) = +\infty$ and $\lim_{x \to 1} f(x) = 0;$

(6) $\lim_{x \to +\infty} f(x) = +\infty,$ $\lim_{x \to -\infty} f(x) = 0$ and $\lim_{x \to 1} f(x) = +\infty;$

(7) $\lim_{x \to +\infty} f(x) = 0,$ $\lim_{x \to -\infty} f(x) = +\infty,$ $\lim_{x \to 1^+} f(x) = -\infty$ and $\lim_{x \to 1^-} f(x) = +\infty;$

(8) $\lim_{x \to +\infty} f(x) = +\infty,$ $\lim_{x \to -\infty} f(x) = 0$ and $\lim_{x \to 1} f(x) = 1;$

Exercise 4. Write the statement of the Sandwich Theorem for $x \to -\infty$.