EXERCISE SHEET 5

WRITTEN SOLUTIONS OF EXERCISES 1.1 AND 2.2 TO BE PRESENTED ON 6/11

Exercise 1. Run the first 5 iterations of the algorithm of the Intermediate Value Theorem for the following functions $f(x)$. Are the hypothesis of the theorem satisfied in every cases? Can you find, by other methods, the exact solution(s) of $f(x) = 0$?

(1) $f(x) = x^2 - 2$ on $[0, 2]$;
(2) $f(x) = x^3 - 2$ on $[0, 2]$;
(3) $f(x) = e^x - 1$ on $[-1.2]$;
(4) $f(x) = e^x - 2$ on $[-1.2]$;
(5) $f(x) = x - \pi$ on $[0, 4]$;
(6) $f(x) = \sin(x)$ on $[0, 20]$;
(7) $f(x) = 1/x$ on $[-1, 2]$;
(8) $f(x) = 1/x$ on $[1, 2]$.
(9) $f(x) = \tan(x)$ on $[-20, 20]$.

Exercise 2. For each of the following functions, determine the domain of definition and whether they are continuous on their domain of definition.

(1) $f(x) = \begin{cases} 1 & \text{if } x \geq 0 \\ -1 & \text{if } x < 0 \end{cases}$

(2) $f(x) = \dfrac{x}{|x|}$

(3) $f(x) = \begin{cases} x & \text{if } x \geq 0 \\ x^2 & \text{if } x < 0 \end{cases}$

(4) $f(x) = \begin{cases} x & \text{if } x \geq 0 \\ x + 1 & \text{if } x < 0 \end{cases}$

(5) $f(x) = \begin{cases} 2x + 1 & \text{if } x \geq 2 \\ -3x + 11 & \text{if } x < 2 \end{cases}$

Exercise 3. What do the following **wrong** definitions of continuity imply for a function $f : \mathbb{R} \to \mathbb{R}$?

(1) $\forall \epsilon > 0 \quad \forall \delta > 0 \quad |x - x_0| < \delta \Rightarrow |f(x) - f(x_0)| < \epsilon$;
(2) $\exists \delta > 0 \quad \forall \epsilon > 0 \quad |x - x_0| < \delta \Rightarrow |f(x) - f(x_0)| < \epsilon$;
(3) $\exists \epsilon > 0 \quad \forall \delta > 0 \quad |x - x_0| < \delta \Rightarrow |f(x) - f(x_0)| < \epsilon$;
(4) $\exists \epsilon > 0 \quad \exists \delta > 0 \quad |x - x_0| < \delta \Rightarrow |f(x) - f(x_0)| < \epsilon$;
(5) $\forall \epsilon > 0 \quad \exists \delta > 0 \quad |f(x) - f(x_0)| < \epsilon \Rightarrow |x - x_0| < \delta$;