# EXERCISE SHEET 6

*WRITTEN SOLUTIONS OF EXERCISES 2.2 AND 3.3 TO BE PRESENTED ON 14/11*

**Exercise 1.** By applying only the definition of derivative, show that for any $a \in \mathbb{R}$ and for any function $f : I \to \mathbb{R}$ differentiable at $x_0 \in \mathbb{R}$,

$$(af)'(x_0) = af'(x_0) .$$

**Exercise 2.** Show the following equalities for the derivatives of the hyperbolic trigonometric functions. Unless otherwise specified, you are allowed to apply all the formulae we have seen in class.

(1) Show that $(\cosh(x))' = \sinh(x)$. (*Hint: apply the definition of hyperbolic sine and cosine*)
(2) Show that $(\sinh(x))' = \cosh(x)$.
(3) Show that $(\cosh^2(x))' = 2\cosh(x)\sinh(x)$.
(4) Show that $(\sinh^2(x))' = 2\cosh(x)\sinh(x)$.
(5) Show that $(\cosh^2(x) - \sinh^2(x))' = 0$, by applying the previous two points. Is this surprising?
(6) Show that $(\cosh(2x))' = 2\sinh(2x)$ and that $(\sinh(2x))' = 2\cosh(2x)$ by using the rule of the derivative of the composition.
(7) Now show again that $(\cosh(2x))' = 2\sinh(2x)$ and that $(\sinh(2x))' = 2\cosh(2x)$, but using the identities $\cosh(2x) = \cosh^2(x) + \sinh^2(x)$ and $\sinh(2x) = 2\sinh(x)\cosh(x)$ and the Leibnitz rule.
(8) Show that $(\tanh(x))' = 1 - \tanh^2(x) = \dfrac{1}{\cosh^2(x)}$.

**Exercise 3.** Compute the derivatives of the following functions, on their domain of definition:

(1) $f(x) = \dfrac{1}{x+1}$
(2) $f(x) = \dfrac{x^2}{x-1}$
(3) $f(x) = \cos(x^2)$
(4) $f(x) = e^{x^2}$
(5) $f(x) = \dfrac{(x+1)^2}{x-1}$
(6) $f(x) = x^3$
(7) $f(x) = \dfrac{1}{1+e^{-x}}$
(8) $f(x) = e^{e^x}$
(9) $f(x) = \cos(x + 2\pi)$
(10) $f(x) = (x^2+1)^3$
(11) $f(x) = \dfrac{\cos(x)}{\sin(x)}$
(12) $f(x) = \sin(e^{2x+1})$
(13) $f(x) = \tan(2x+1)$
(14) $f(x) = \tanh(x^3)$
(15) $f(x) = (\tanh(x))^3$