## **EXERCISE SHEET 9**

## WRITTEN SOLUTIONS OF EXERCISES 1.2 AND 1.15 TO BE PRESENTED ON 4/12

**Exercise 1.** For each of the following functions f, determine the intervals on which f is monotone increasing, monotone decreasing, the local minima and the local maxima of f. Sketch a picture of the graph of f. (*Hint: do not forget to consider the domain of definition and the limits of f*).

$\begin{array}{ll} (2) \ f(x) = -x^2 + x. \\ (3) \ f(x) = 2x^2 + x - 6. \\ (4) \ f(x) = 4x^3 + 3x^2 - 6x + 2. \\ (5) \ f(x) = 1 - 12x - 9x^2 - 2x^3. \\ (6) \ f(x) = 3 - 2x^2 + 4x^4. \\ (7) \ f(x) = \frac{2x}{x^2 + 1}. \\ (8) \ f(x) = \sinh(x). \\ (9) \ f(x) = \cosh(x). \\ (10) \ f(x) = \frac{1}{\cosh(x)}. \\ (11) \ f(x) = \tanh(x). \end{array}$ $\begin{array}{ll} (13) \ f(x) = \frac{x^2}{2 - x}. \\ (14) \ f(x) = \frac{3x + 2}{1 - x^2}. \\ (15) \ f(x) = \frac{3x + 1}{1 - x^2}. \\ (15) \ f(x) = \frac{x^3}{x + 1}. \\ (16) \ f(x) = \sqrt{1 - x}. \\ (17) \ f(x) = x + e^{-x}. \\ (18) \ f(x) = \ln(1 - x). \\ (19) \ f(x) = \ln(x^2 - 4). \\ (10) \ f(x) = \tanh(x). \end{array}$	(1) $f(x) = 6x + 2$ .	(12) $f(x) = \frac{x-1}{x^2}$ .
(b) $f(x) = 2x + x = 0$ . (c) $f(x) = 2x + x = 0$ . (d) $f(x) = 4x^3 + 3x^2 - 6x + 2$ . (e) $f(x) = 1 - 12x - 9x^2 - 2x^3$ . (f) $f(x) = 3 - 2x^2 + 4x^4$ . (f) $f(x) = \frac{2x}{x^2 + 1}$ . (g) $f(x) = \sinh(x)$ . (g) $f(x) = \cosh(x)$ . (h) $f(x) = \frac{1}{\cosh(x)}$ . (h) $f(x) = x \ln(x)$ .		(13) $f(x) = \frac{x^2}{2-x}$ .
$\begin{array}{ll} (5) \ f(x) = 1 - 12x - 9x^2 - 2x^3. \\ (6) \ f(x) = 3 - 2x^2 + 4x^4. \\ (7) \ f(x) = \frac{2x}{x^2 + 1}. \\ (8) \ f(x) = \sinh(x). \\ (9) \ f(x) = \cosh(x). \\ (10) \ f(x) = \frac{1}{\cosh(x)}. \end{array}$ $\begin{array}{ll} (15) \ f(x) = \frac{x^3}{x + 1}. \\ (16) \ f(x) = \sqrt{1 - x}. \\ (17) \ f(x) = x + e^{-x}. \\ (18) \ f(x) = \ln(1 - x). \\ (19) \ f(x) = \ln(x^2 - 4). \\ (20) \ f(x) = x \ln(x). \end{array}$		- ~
(6) $f(x) = 0$ $2x + 1x$ (16) $f(x) = \sqrt{1-x}$ . (7) $f(x) = \frac{2x}{x^2 + 1}$ . (8) $f(x) = \sinh(x)$ . (9) $f(x) = \cosh(x)$ . (10) $f(x) = \frac{1}{\cosh(x)}$ . (12) $f(x) = x + e^{-x}$ . (13) $f(x) = x + e^{-x}$ . (14) $f(x) = x + e^{-x}$ . (15) $f(x) = x + e^{-x}$ . (16) $f(x) = x + e^{-x}$ . (17) $f(x) = x + e^{-x}$ . (18) $f(x) = \ln(1-x)$ . (19) $f(x) = \ln(x^2 - 4)$ . (20) $f(x) = x \ln(x)$ .		
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	(10) $f(x) = \frac{1}{\cosh(x)}.$ (11) $f(x) = \tanh(x).$	(20) $f(x) = x \ln(x)$ .

**Exercise 2.** Let  $f : \mathbb{R} \to \mathbb{R}$  be any function, and let  $g(x) = e^{f(x)}$ .

- (1) Supposing that *f* is differentiable, show that for every  $x \in \mathbb{R}$ , f'(x) > 0 if and only if g'(x) > 0.
- (2) Show that f and g have the same local minima and maxima.
- (3) Now show, applying the definition of monotonicity, that *f* is monotone increasing if and only if *g* is monotone increasing. (*Warning: we are not supposing that f is differentiable.*)
- (4) Finally, show that for every f and g monotone increasing,  $f \circ g$  is monotone increasing.

**Exercise 3.** Show that, if  $f : I \to \mathbb{R}$  is a monotone decreasing function, then  $f' \leq 0$ . Find a function f which is strictly decreasing but f' is not strictly negative.