Exercice 1

1) On a
$$f_1(0) = f_1(\pi) = 0$$
$$f_2(0) = \cos\frac{\pi}{2} = 0$$
$$f_2(\pi) = \cos\left(-\frac{\pi}{2}\right) = 0$$
$$f_3(0) = f_3(\pi) = 0$$
$$f_4(0) = f_4(\pi) = 0$$

2) Calculons les solutions avec données initiales :

$$\begin{cases} u(x,0) = f_1(x) \\ u_t(x,0) = 0 \end{cases} \longrightarrow u_{10}(x,t) = 3\cos(4at)\sin(4x)$$

$$\begin{cases} u(x,0) = 0 \\ u_t(x,0) = f_1(x) \end{cases} \longrightarrow u_{01}(x,t) = \frac{3}{4a}\sin(4at)\sin(4x)$$

Rmq : $\cos\left(\frac{\pi}{2} - x\right) = \sin x$

$$\begin{cases} u(x,0) = f_2(x) \\ u_t(x,0) = 0 \end{cases} \longrightarrow u_{20}(x,t) = \cos(at)\sin(x)$$

$$\begin{cases} u(x,0) = 0 \\ u_t(x,0) = f_2(x) \end{cases} \longrightarrow u_{02}(x,t) = \frac{1}{a}\sin(at)\sin(x)$$

$$\begin{cases} u(x,0) = f_3(x) \\ u_t(x,0) = 0 \end{cases} \longrightarrow u_{30}(x,t) = \cos(3at)\sin(3x) + 7\cos(5at)\sin(5x)$$

$$\begin{cases} u(x,0) = 0 \\ u_t(x,0) = f_3(x) \end{cases} \longrightarrow u_{03}(x,t) = \frac{1}{3a}\sin(3at)\cos(3x) + \frac{7}{5a}\sin(5at)\sin(5x)$$

Rmq : $4\sin(x)\cos(x) = 2\sin(2x)$

$$\begin{cases} u(x,0) = f_4(x) \\ u_t(x,0) = 0 \end{cases} \longrightarrow u_{40}(x,t) = 2\cos(2at)\sin(2x)$$

$$\begin{cases} u(x,0) = 0 \\ u_t(x,0) = f_4(x) \end{cases} \longrightarrow u_{04}(x,t) = \frac{1}{a}\sin(2at)\cos(2x)$$

Alors la solution avec donnés initials

$$\begin{cases} u(x,0) = f_i(x) \\ u_t(x,0) = f_j(x) \end{cases}$$

est la fonction $u_{i0}(x,t) + u_{0j}(x,t)$.